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Persistence at the onset of spatiotemporal intermittency in coupled map lattices

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Abstract. – We study persistence in coupled circle map lattices at the onset of spatiotemporal intermittency, an onset which marks a continuous transition, in the universality class of directed percolation, to a unique absorbing state. We obtain a local persistence exponent of $\theta_l = 1.49 \pm 0.02$ at this transition, a value which closely matches values for θ_l obtained in stochastic models of directed percolation. This result constitutes suggestive evidence for the universality of persistence exponents at the directed percolation transition. Given that many experimental systems are modelled accurately by coupled map lattices, experimental measurements of this persistence exponent may be feasible.

The study of extended dynamical systems is relevant to the understanding of many phenomena in condensed matter physics, such as pattern formation and non-equilibrium phase transitions in coupled chemical reactions, charge density waves and Josephson junction arrays [1]. A simple model which captures much of the underlying complexity of these systems is the coupled map lattice, defined as a collection of elements on a lattice which locally exhibit chaotic dynamics, together with a diffusive local coupling between these elements [2]. Coupled map lattices exhibit a remarkable variety of behaviour, ranging from periodic spatio-temporal structure to intermittency and chaos.

Spatially extended dynamical systems exhibiting spatio-temporal intermittency possess both laminar and turbulent phases. The laminar phase is characterized by periodic or even weakly chaotic dynamics, while no spatio-temporally regular structure exists in the turbulent regime. Spatio-temporal intermittency refers to the properties of the state which marks the transition between laminar and turbulent phases. A typical spatial pattern arising in spatio-temporal intermittency consists of fluctuating domains of laminar regions interspersed amidst turbulent ones. Such intermittency is often the precursor of fully developed chaos [2].

An initially laminar site becomes turbulent only if at least one of its neighbours was turbulent at the previous time. A turbulent site can either relax spontaneously to a laminar

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state or contaminate its neighbours at the next time step [3]. In some regimes of parameter space, turbulent states percolate in space-time, leading to an active phase. In other regimes, turbulent states die out, following which the system is confined to the laminar (equivalently, inactive) state for all time. This laminar state is thus a unique absorbing state.

It has been conjectured that critical exponents associated with the onset of spatiotemporal intermittency belong generically to the universality class of directed percolation [4]. Underlying this conjectured correspondence is the following more general question: Are transitions in deterministic chaotic systems governed by the universality classes of stochastic models [5]? To address this issue, we study persistence and persistence exponents in coupled map lattices at the onset of spatiotemporal intermittency. Persistence exponents for stochastic systems have attracted attention recently as they appear to be a new class of exponents, not derivable in general from the other static and dynamic exponents [6]. Systems which exhibit a non-trivial persistence exponent include Ising and Potts models in one and higher dimensions, the diffusion equation, the random velocity and random acceleration problems and several models for interface growth [7].

Our principal result is the following: We find that the local persistence exponent θ_l at the onset of spatiotemporal intermittency exhibits a remarkable universality at the critical points we have examined. The value of the persistence exponent is $\theta_l = 1.49 \pm 0.02$, in close agreement with values obtained for stochastic models with discrete states in one dimension, at the directed percolation transition [9,8,10]. Our model, however, differs from these earlier models in that it is *not* stochastic but deterministic. Another important difference is that the local variable in our case is a *continuum* variable, unlike in the models studied earlier.

This result is unusual for the following reason: The persistence exponent is known to be far less universal than the other three exponents which typically characterize scaling behaviour in interacting stochastic systems, the static exponents η and ν and the dynamical exponent z. The reason is that persistence probes the full, in general *non-Markovian*, time evolution of a local fluctuating variable, such as a spin or density field, from its initial state [7]. Knowing the asymptotic properties of the evolution kernel governing this time evolution is insufficient to obtain the persistence exponent, although in many cases progress can be made through controlled expansions about Markov processes [7]. Thus, such universality of the persistence exponent at the directed percolation transition, in particular for systems with deterministic chaos, is *a priori* unexpected.

We study the dynamics of a 1-dimensional coupled map lattice, with on-site circle maps coupled diffusively to nearest neighbours [2,11]:

$$x_{i,t+1} = f(x_{i,t}) + \frac{\epsilon}{2}(f(x_{i-1,t}) + f(x_{i+1,t}) - 2f(x_{i,t})) \mod 1 \tag{1}$$

where t is the discrete time index, and i is the site index: $i = 1, \ldots L$, with L the system size. The parameter ϵ measures the strength of the diffusive coupling between site i and its two neighbours. The on-site map is chosen to be

$$f(x) = x + \omega - \frac{k}{2\pi}\sin(2\pi x) \tag{2}$$

where the parameter k denotes the nonlinearity. All sites are updated in parallel.

The synchronized spatiotemporal fixed point $x^\star$ corresponds to the unique absorbing state. Turbulent sites are those which have local variables which differ from $x^\star$. The fixed point solution $x^\star$ is easily obtained:

$$x^\star = \frac{1}{2\pi}\sin^{-1}(\frac{2\pi\omega}{k}). \tag{3}$$
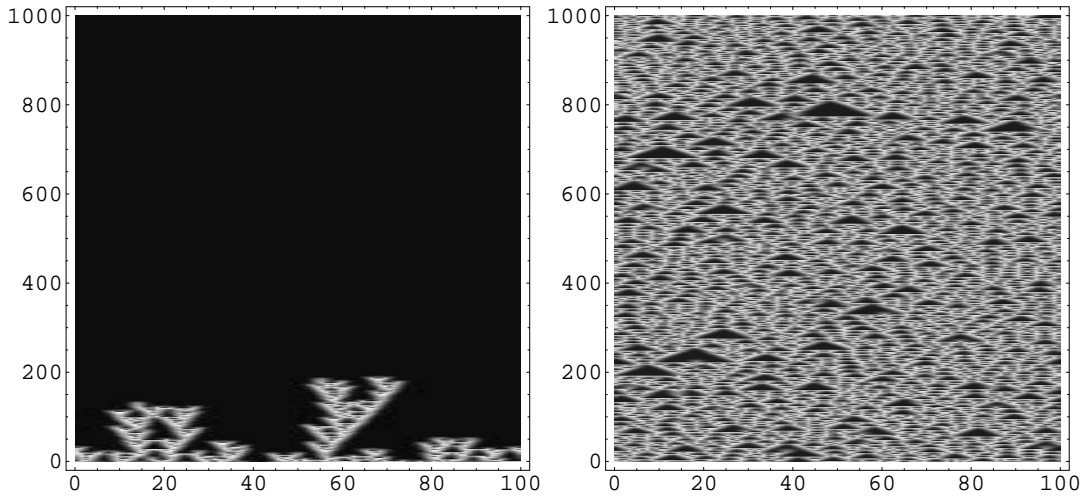
Fig. 1 – Time evolution of the coupled circle map lattice defined through eqns. 1-2, in a system of size $L = 100$, for parameters $k = 1$ and $\epsilon = 0.63775$. Figure 1(a) is for $\omega = 0.05$ (laminar region) while fig. 1(b) is for $\omega = 0.1$ (turbulent region). The horizontal axis is the site index $i = 1, \ldots L$; the vertical axis denotes discrete time t.

Figures 1(a),1(b), and 2(a) show space-time density plots of x_i for three choices of parameter values. The site index is plotted along the x axis and time evolves along the y direction. Laminar regions are dark in this representation. The first of these plots, fig. 1(a), is for parameter values for which the final state is laminar, while the second, fig. 1(b), is for parameter values in the chaotic regime. Fig. 2(a) marks the transition between laminar and chaotic regimes.

The critical exponents of this coupled map lattice at the onset of spatio-temporal intermittency have been obtained numerically; these match directed percolation values to high accuracy [12]. This model, at the critical points studied in this paper, constitutes the *only* known example of a coupled map lattice with a *unique* absorbing state, whose transition falls cleanly in the universality class of directed percolation [12,13]. Finding direct transitions to a unique absorbing state from turbulent states in coupled map lattices is nontrivial; transitions into spatio-temporally periodic patterns are far more common. As a consequence, even though the past decade has seen considerable activity in this field, we know of no other examples of such critical points in the literature [14].

A further advantage of this model system is the absence of any additional special spatio-temporal structures, a feature which commonly disrupts the analogy to DP in extended dynamical systems [15]. Operationally, this means that no asynchronicity in updates need be introduced here to destroy "solitonic" behaviour [15], Thus, coupled circle maps are clean model systems for checking DP universality.

Persistence in the context of stochastic processes is defined as the probability $P(t)$ that a stochastically fluctuating variable has not crossed a threshold value upto time t [6]. For a spin system with discrete states, such as Ising or Potts spins, persistence is defined in terms of the probability that a given spin has not flipped out of its initial state upto time t. A power-law
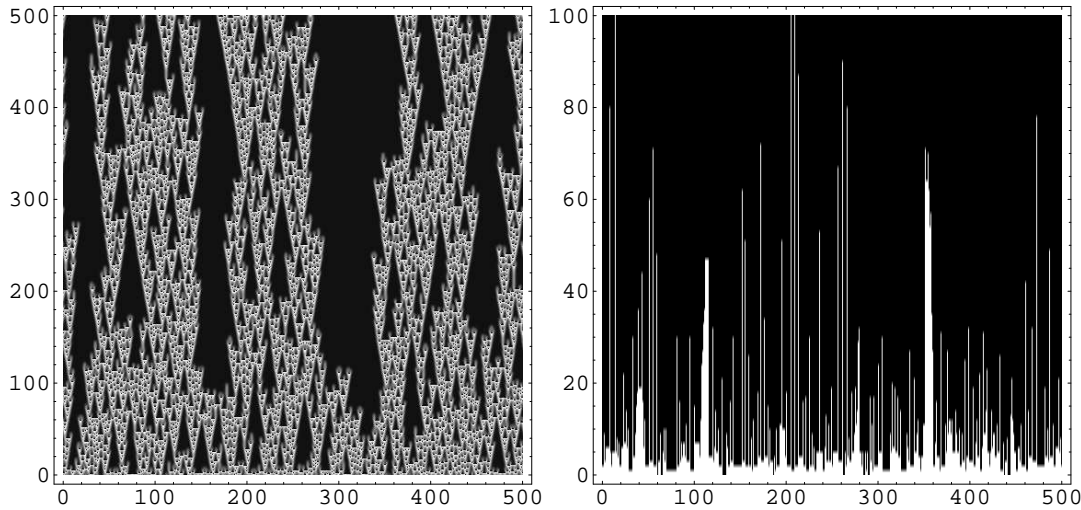
Fig. 2 – Time evolution of the coupled circle map lattice defined through eqns. 1-2, in a system of size $L = 500$ at the critical point ($k = 1$, $\omega = 0.068$, $\epsilon = 0.63775$). The horizontal axis is the site index $i = 1, \ldots L$ and the vertical axis denotes discrete time t. Fig. 2(a) is obtained from a density plot of the actual $x_{i,t}$ values (the absorbing regions appear dark). Fig. 2(b) is a plot of the persistent sites (marked in white) *vs.* time.

tail to $P(t)$, *i.e.*

$$P(t) \sim 1/t^{\theta}, \tag{4}$$

defines the persistence exponent θ, with $P(t)$ averaged over an ensemble of random initial conditions.

What is the appropriate description of persistence in coupled map lattices? A natural generalization of the ideas above *defines* local persistence in terms of the probability that a local state variable $x_{i,t}$ does not cross the fixed point value $x^{\star}$ up to time t. With this definition, we study persistence in the coupled map lattice defined through Eqn. (1) *via* simulations on systems of linear size ranging from $L = 10$ to $L = 10^5$. We start with the local variable $x_{i,0}$ distributed uniformly in the interval $[0, 1]$, update all sites synchronously, and measure the normalized fraction of persistent sites, defined as sites for which $x_{i,t}$ has not *crossed* $x^{\star}$ up to time t. Thus, sites for which the sign of $(x_{i,t} - x^{\star})$ has not changed upto time t are persistent at time t.

Fig. 2(b) shows the persistent sites at the critical point corresponding to the configuration of fig. 2(a). The inset to Fig. 3 shows plots of $P(t)$ at three different parameter values, illustrating the following points: (a) $P(t)$ in the laminar region saturates to a time-independent value P_{∞} at large times, with P_{∞} becoming smaller and smaller as the transition is approached; (b) Precisely at the transition between chaotic and laminar regimes, P_{∞} is zero and $P(t)$ decays as a power law, thereby defining the local persistence exponent θ_l and finally; (c) In the chaotic regime $P(t)$ decays exponentially. These curves are analogous to data obtained for persistence in the Domany-Kinzel cellular automaton [9], with the identification of the turbulent phase here with the active phase of the Domany-Kinzel model and the laminar phase with the inactive one.

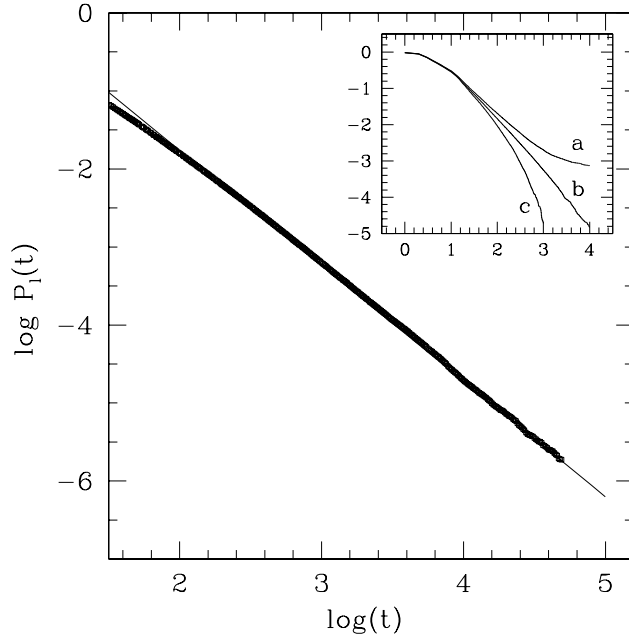Figure 3 (main panel) shows our most accurate determination of the local persistence

Fig. 3 – Local persistence distribution $P_l(t)$ vs time t for the critical point at: $k = 1$, $\omega = 0.068$, $\epsilon = 0.63775$. Inset shows the log-log plot of $P_l(t)$ vs t for parameters $k = 1$, $\epsilon = 0.63775$ and (a) $\omega = 0.066$ (below critical point); (b) $\omega = 0.068$ (at the critical point displaying power law scaling) and (c) $\omega = 0.07$ (above critical point).

probability distribution $P_l(t)$ at a critical point, on a system of size $L = 10^5$. We work at the two critical points known to high accuracy ($k = 1.0, \omega = 0.068, \epsilon = 0.63775$) and ($k = 1.0, \omega = 0.064, \epsilon = 0.73277$), averaging over 10^4 initial conditions. To within the accuracy of our simulations, the values of θ_l at both these critical points are the same, consistent with a persistence exponent of $\theta_l = 1.49 \pm 0.02$ [16].

If the diverging length (and time) scales at the continuous directed percolation transition determine the scaling properties of local persistence, we expect a scaling form of the type

$$P_l(t, L, \delta) \sim t^{-\theta_l} \mathcal{F}_\pm(\delta^{\nu_\parallel} t, L^{-z} t). \qquad (5)$$

Here $z = \nu_\parallel/\nu_\perp$, with $\nu_\perp$ the standard 1+1 dimensional DP exponent governing the decay of correlation lengths in the spatial direction and $\nu_\parallel$ the analogous exponent for the time direction ($\nu_\parallel = 1.733847, \nu_\perp = 1.096854$) [9, 17]. $\mathcal{F}_\pm$ is a universal scaling function (the subscript $\pm$ refers to the direction of approach to the critical point), while δ measures the distance from the critical point. At $\delta = 0$, $\mathcal{F}_\pm$ behaves asymptotically as $\mathcal{F}_\pm(0,0) \sim$ const; $\mathcal{F}_\pm(x,0) \sim x^{\theta_l}, x \to \infty$; $\mathcal{F}_\pm(0,y) =$ const, $y \ll 1$ and $\mathcal{F}_\pm(0,y) \sim y^{\theta_l}, y > 1$. This scaling function is similar to one proposed in the study of the spatial scaling properties of persistence [18]. Fig. 5 shows this scaling function at criticality ($\delta = 0$), for various system sizes L. The best data collapse is obtained for $z = 1.58$ and $\theta_l = 1.485$. The quality of the data collapse verifies that the scaling forms are fully consistent with directed percolation exponents [8, 9].
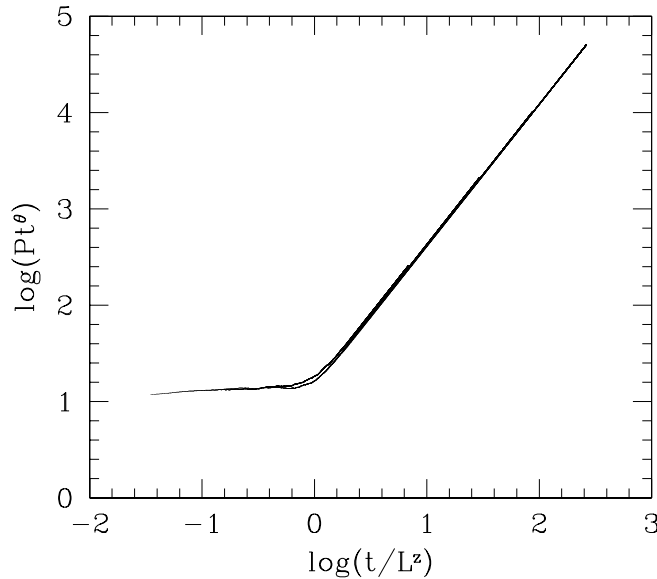
Fig. 4 – Log-log plot of scaled persistence probability $P_l(t)t^{\theta_l}$ versus scaled time tL^{-z} for different lattice sizes at the critical point: $k = 1$, $\omega = 0.068$, $\epsilon = 0.63775$. The data collapses onto a single curve for $L = 50, 100, 300, 500$ and 1000.

Our result for the local persistence exponent in coupled circle maps at the laminar-turbulent transition agrees well with the best estimates for θ in the Domany-Kinzel automaton at the directed percolation transition in $1+1$ dimensions, where Hinrichsen and Koduvely found $\theta = 1.5 \pm 0.02$ [9]. A recent study by Albano and Muñoz finds $\theta = 1.5 \pm 0.01$ for the contact process at its critical point in $1+1$ dimensions [10], a value in agreement with our simulations. These papers also study *global* persistence properties [19], drawing on calculations of the global persistence exponent θ_g for phase ordering in Ising and Potts systems, where such exponents differ from their local values. However, the properties of θ_g remain controversial for DP-related problems [20]. Given the somewhat contrived definition of global persistence in directed percolation, we believe that it would certainly be far harder to access this exponent experimentally than the local exponent.

In summary, this Letter presents suggestive numerical evidence for the universality of the persistence exponent at the onset of spatio-temporal intermittency in coupled map lattices. We obtain a persistence exponent whose value is numerically identical to values obtained at the directed percolation transition in $1+1$ dimensions in a variety of systems, including the Domany-Kinzel automaton and the one-dimensional contact process [21]. However, the model we study here is both *deterministic* and possesses a *continuum* degree of freedom at each lattice site, unlike these other models.

The implications of these results for experiments is particularly noteworthy. Spatio-temporal intermittency is a common phenomena of many extended systems. It is seen in experiments on convection [22] and in the "printers instability" [23]. The state variables of coupled map lattices can often be identified with physical quantities such as voltages, cur-

rents, pressures, temperatures, concentrations or velocities. A recent experiment by Rupp, Richter and Rehberg [24] obtains directed percolation exponents at the transition to spatio-temporal intermittency in a one-dimensional system of ferrofluid spikes, driven by an external oscillating magnetic field. We believe that obtaining the local persistence exponent from an experiment such as this should be fairly straightforward. We hope this Letter stimulates further work towards identifying new candidates for measuring persistence exponents in real world phenomena.

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